

UNIT – V : DECISION THEORY

Syllabus: Decision theory: Risk and uncertainty in decision-making – minimax, maximin and regret criterion – Hurwicz and Laplace criteria in decision making – decision tree analysis.

What is Decision-making?

- Decision making is an integral part of management planning, organizing, controlling processes.
- The decision maker selects one strategy (course of action) over others depending on some criteria, like utility, sales, cost or rate of return.

Types of Decisions

- Strategic Decision – Concerned with external environment of the organization.
- Administrative Decision – Concerned with structuring and acquisition of the organization's resources so as to optimize the performance of the organization.
- Operating Decision – Concerned with day to day operations of the organization such as pricing, production scheduling, inventory levels, etc.

Terminologies

- Course of action: refers to the decision alternatives or actions that is been chosen by the decision-maker.
- States of nature: Exhaustive list of possible future events/alternatives. Decision maker has no direct control over the occurrence of particular event.
- Payoff: Effectiveness associated with specified combination of a course of action and state of nature. Also known as profits or conditional values.

• **TYPES OF DECISION MAKING ENVIRONMENTS**

- DECISION MAKING UNDER CERTAINTY
- DECISION MAKING UNDER UNCERTAINTY
- DECISION MAKING UNDER RISK

SIX STEPS IN DECISION MAKING :

- Clearly define the problem in hand
- List the possible alternatives
- Identify the possible outcomes or states of nature
- List the payoff or profit of each combination of alternatives & outcomes
- Select one of the mathematical decision theory models
- Apply the model and make your decision

DECISION THEORY

□ **DECISION MAKING UNDER CERTAINTY** : Decision makers know with certainty the consequence of every alternative or decision choice. Naturally they will choose the alternative that will result in the best outcome. **Example is making a fixed deposit in a bank.**

□ **DECISION MAKING UNDER UNCERTAINTY** :

Several criteria exist for making decision under these conditions :

1. Maximax (optimistic)
2. Maximin (pessimistic)
3. Criterion of realism (Hurwicz)
4. Equally likely (Laplace)
5. Minimax regret

DECISION THEORY

A CASE STUDY

- John Thompson is the President of Stewarts & Lloyds of India ltd.
 - John Thompson's problem is to identify whether to expand his product line by manufacturing and marketing a **new product : washing machine**.
 - In order to make a proposal for submitting to his board of directors, Thompson thought of following three alternatives that are available to him.
1. **To construct a large new plant to manufacture the washing machine**
 2. **To construct a small plant to manufacture the washing machine**
 3. **No plant at all** (that is he has the option of not developing the new product line).

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- Thompson determines that there are only **two possible state of natures** :
 1. The market for the washing machine could be favorable, meaning that there is a high demand for the product
 2. it could be unfavorable, meaning that there is a low demand for the washing machine.

John Thompson evaluated the profits associated with various outcomes. He thinks :

- ❑ With a favorable market, a large facility would result in a profit of Rs.2,00,000 to his firm. But Rs.2,00,000 is a **conditional value** because Thompson's receiving the money is **conditional upon both his building a large factory and having a good market.**
- ❑ The large facility and unfavorable market would result in net loss of Rs.1,80,000.

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- ❑ A small plant with a favorable market would result in a net profit of Rs.100,000 .
- ❑ A small plant with unfavorable market would result in a net loss of Rs.20,000 .
- ❑ doing nothing, that is neither to make large facility nor a small plant, in either market would result in no profits.

The decision table or payoff table for Thompson's conditional values is shown in table on next slide .

DECISION THEORY

ALTERNATIVES	STATES OF NATURE	
	Favorable Market (Rs.)	Unfavorable Market (Rs.)
Construct a Large plant	200,000	- 180,000
Construct a Small plant	100,000	- 20,000
Do nothing	0	0

DECISION THEORY

MAXIMAX : The maximax criterion is used to find the alternative that maximises the maximum payoff. First locate the maximum payoff for each alternative, and then pick that alternative with the maximum number. It locates the alternative with the highest possible gain : therefore it is called an **optimistic decision criteria**. Thompson’s maximax choice is the first alternative “**construct a large plant**”.

TABLE 1 : THOMPSON’S MAXIMAX DECISION

ALTERNATIVES	STATES OF NATURE		MAXIMUM IN A ROW (Rs.)
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	200,000	-180,000	200,000 MAXIMAX
Construct a small plant	100,000	-20,000	100,000
Do nothing	0	0	0

DECISION THEORY

MAXIMIN : The maximin criterion is used to find the alternative that maximises the minimum payoff or consequence for every alternative. First locate the minimum payoff for each alternative and then pick that alternative with the maximum payoff. This decision criterion locates the alternative that gives the **best of the worst (minimum) payoffs**, and thus it is called a **pessimistic decision criterion**. This criterion guarantees that the payoff **WILL BE AT LEAST THE MAXIMIN VALUE**. Thompson's maximin choice is **“do nothing”**.

TABLE 2 : THOMPSON'S MAXIMIN DECISION

ALTERNATIVES	STATES OF NATURE		MINIMUM IN A ROW (Rs.)
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	200,000	- 180,000	- 180,000
Construct a small plant	100,000	- 20,000	- 20,000
Do nothing	0	0	0 MAXIMIN ₁₂

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CRITERION OF REALISM (HURWICZ CRITERION)

The criterion of realism is a compromise between an optimistic and a pessimistic decision. A **coefficient of realism (α)** is used to measure the degree of optimism of the decision maker. this coefficient, α lies between 0 and 1. The weighted average is computed as follows:

Weighted average = (α) x (maximum in row) + (1 - α) x (minimum in row)

In the given case, John Thompson sets $\alpha = 0.80$ and thus the best decision would be to **construct a large plant** as shown in **table 3** below.

TABLE 3 : THOMPSON'S CRITERION OF REALISM DECISION

ALTERNATIVES	STATES OF NATURE		CRITERON OF REALISM OR WEIGHTED AVERAGE ($\alpha = 0.80$)
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	200,000	- 180,000	Rs. 1,24,000 REALISM
Construct a small plant	100,000	-20,000	76,000
Do nothing	0	0	0

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EQUALLY LIKELY (LAPLACE) :

This criterion uses all the payoffs for each alternative . this is also called laplace, decision criterion. This criteria finds the average payoff for each alternative and select the alternative with highest average. This criterion assumes that all probability of occurrence for the state of natures are equal, and thus each state of nature is equally likely. Thompson’s choice as per this criterion is the second alternative, “construct a small plant”.

TABLE 4 : THOMPSON’S EQUALLY LIKELY DECISION

ALTERNATIVES	STATES OF NATURE		ROW AVERAGE (Rs.)
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	200,000	- 180,000	10,000
Construct a small plant	100,000	- 20,000	40,000 EQUALLY LIKELY
Do nothing	0	0	0

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MINIMAX REGRET

This decision criterion is based on opportunity loss or regret. The opportunity loss or regret is the **amount lost by not picking the best alternative in a given state of nature.** The first step is to create the opportunity loss table. **Opportunity loss** for any state of nature, or any column, **is calculated by subtracting each payoff in the column from the best payoff in the same column.**

Thompson's opportunity loss table is shown in **table 5.** Using the opportunity loss table, the minimax regret criterion **finds the alternative that minimises the maximum opportunity loss within each alternative.** First find the maximum opportunity loss for each alternative. Next, looking at these maximum values, pick that alternative with minimum number. We can see that **minimax regret choice is the second alternative, "construct a small plant".**

TABLE 5 : OPPORTUNITY LOSS TABLE

ALTERNATIVES	STATES OF NATURE		MAXIMUM IN A ROW
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	0 [200,000 – 200,000]	180,000 [0 - (- 180,000)]	180,000
Construct a small plant	100,000 [200,000 – 100,000]	20,000 [0 - (- 20,000)]	100,000 MINIMAX
Do nothing	200,000 [200,000 - 0]	0 [0 - 0]	200,000

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❑ DECISION MAKING UNDER RISK

Decision making under risk is a **decision situation in which several possible state of nature may occur and the probabilities of these states of nature are known**. The decision under risk are taken based on following :

- ❑ Expected monetary value or expected value (**EMV**)
- ❑ Expected value of perfect information (**EVPI**)
- ❑ Expected opportunity loss (**EOL**)

EXPECTED MONETARY VALUE

The expected monetary value (**EMV**) for an alternative is just the sum of products of payoffs and probability of each state of nature.

$$\begin{aligned} \text{EMV (Alternative ,i)} = & \text{(Payoff of first state of nature) x (Probability of first state of} \\ & \text{nature)} \\ & + \text{(Payoff of second state of nature) x (Probability of second state of} \\ & \text{nature)} \\ & + \text{(Payoff of third state of nature) x (Probability of third state of nature)} \\ & + \dots\dots\dots + \text{(Pay of last state of nature) x (Probability of last state of nature)} \end{aligned}$$

The alternative with maximum **EMV** is then chosen.

Suppose Thompson now believes that the **probability of a favorable market is exactly the same as the probability of an unfavorable market**.

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SOLUTION BY USING EMV METHOD :

$$\text{EMV (Large Plant)} = (0.50) \times (\text{Rs.}200,000) + (0.50) \times (-\text{Rs.}180,000) = \text{Rs.}10,000$$

$$\text{EMV (Small Plant)} = (0.50) \times (\text{Rs.}100,000) + (0.50) \times (-\text{Rs.}20,000) = \text{Rs.}40,000$$

$$\text{EMV (Do Nothing)} = (0.50) \times (\text{Rs.}0) + (0.50) \times (\text{Rs.}0) = \text{Rs.}0$$

THE LARGEST EXPECTED VALUE OF Rs.40,000 results from the second alternative “construct a small plant”. Thus Thompson would proceed to set up small plant.

TABLE 6 : Decision Table with Probabilities and EMVs for John Thompson

Alternatives	States of Nature		EMV
	Favorable Market	Unfavorable Market	
Construct Large Plant	200,000	- 180,000	10,000
Construct Small Plant	100,000	- 20,000	40,000 Maximum Value
Do Nothing	0	0	0
Probability	0.50	0.50	

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❑ EXPECTED VALUE OF PERFECT INFORMATION (EVPI)

Suppose, John Thompson has been approached by a marketing consultant that they are willing to help John with some perfect information whether the market is favorable for the proposed product enabling John to take correct decision and prevent him from making a very expensive mistake. **Marketing consultant would charge John Thompson Rs.65,000 for providing such information. What John should do in this situation ?**

- 1. Should he hire the marketing consultant for making the market study ?**
- 2. Even if the information provided is perfectly accurate, is it worth to pay Rs.65,000 to marketing consultant?**
- 3.If not, what would it be worth ?**

In this case, two related terms are investigated :

1. The expected value of perfect information (**EVPI**), and
2. The expected value with perfect information (**EVwPI**)

$$\begin{aligned} \text{EVwPI} = & \text{(Best payoff for first state of nature)} \times \text{(Probability of first state of nature)} + \\ & \text{(Best payoff for second state of nature)} \times \text{(Probability for second state of} \\ & \text{nature)} \\ & + \dots\dots\dots + \text{(Best payoff for last state of nature)} \times \text{(Probability for last state of} \\ & \text{nature)} \end{aligned}$$

$$\text{EVPI} = \text{EVwPI} - \text{Maximum EMV}$$

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EVPI With respect to **table 6** is calculated as follows :

1. The best payoff for the state of nature “favorable market” is **Rs.200,000**. The best payoff for the state of nature “unfavorable market” is **Rs.0**.

Now, EVwPI = (Rs.200,000) (0.50) + (Rs.0) (0.50) = Rs.100,000

Thus, if John had perfect information, the payoff would average **Rs.100,000**

2. The maximum **EMV** without additional information is **Rs.40,000**(from table 6)

So, EVPI = (Expected value with perfect information) – (Maximum EMV)
= Rs.100,000 - Rs.40,000 = Rs.60,000

Thus, at best John Thompson would be willing to pay for perfect information is Rs.60,000 based on assumption that the probability of each state of nature is 0.50.

❑ EXPECTED OPPORTUNITY LOSS

An alternative approach is to maximise **EMV** by **minimising expected opportunity loss**. First an opportunity loss table is constructed. Then the **EOL** is computed for each alternative by multiplying the opportunity loss by the probability and adding these together.

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Using **table 5**, we compute the **EOL** for each alternative as follows :

$$\mathbf{EOL(Construct\ Large\ plant) = (0.50) (Rs.0) + (0.50) (Rs.180,000) = Rs.90,000}$$

$$\mathbf{EOL (Construct\ a\ small\ plant) = (0.50) (Rs.100,000) + (0.50) (Rs.20,000) = Rs.60,000}$$

$$\mathbf{EOL (Do\ nothing) = (0.50) (Rs.200,000) + (0.50) (Rs.0) = Rs.100,000}$$

From the **EOL Table 7** , we see that the best decision would be the second alternative :
 “construct a small plant”.

TABLE 7 : EOL TABLE FOR JOHN THOMPSON

ALTERNATIVES	STATES OF NATURE		EOL
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	0	180,000	90,000
Construct a small plant	100,000	20,000	60,000 Minimum Value
Do nothing	200,000	0	100,000
Probabilities	0.50	0.50	

DECISION TREES

Any problem that can be presented in a decision table can also be graphically illustrated by a decision tree. All decision trees contains **decision nodes and state of nature nodes**.

- ❑ **Decision nodes** are represented by squares from which one or several alternatives may be chosen
- ❑ **State-of-nature nodes** are represented by circles out of which one or more state-of-nature will occur

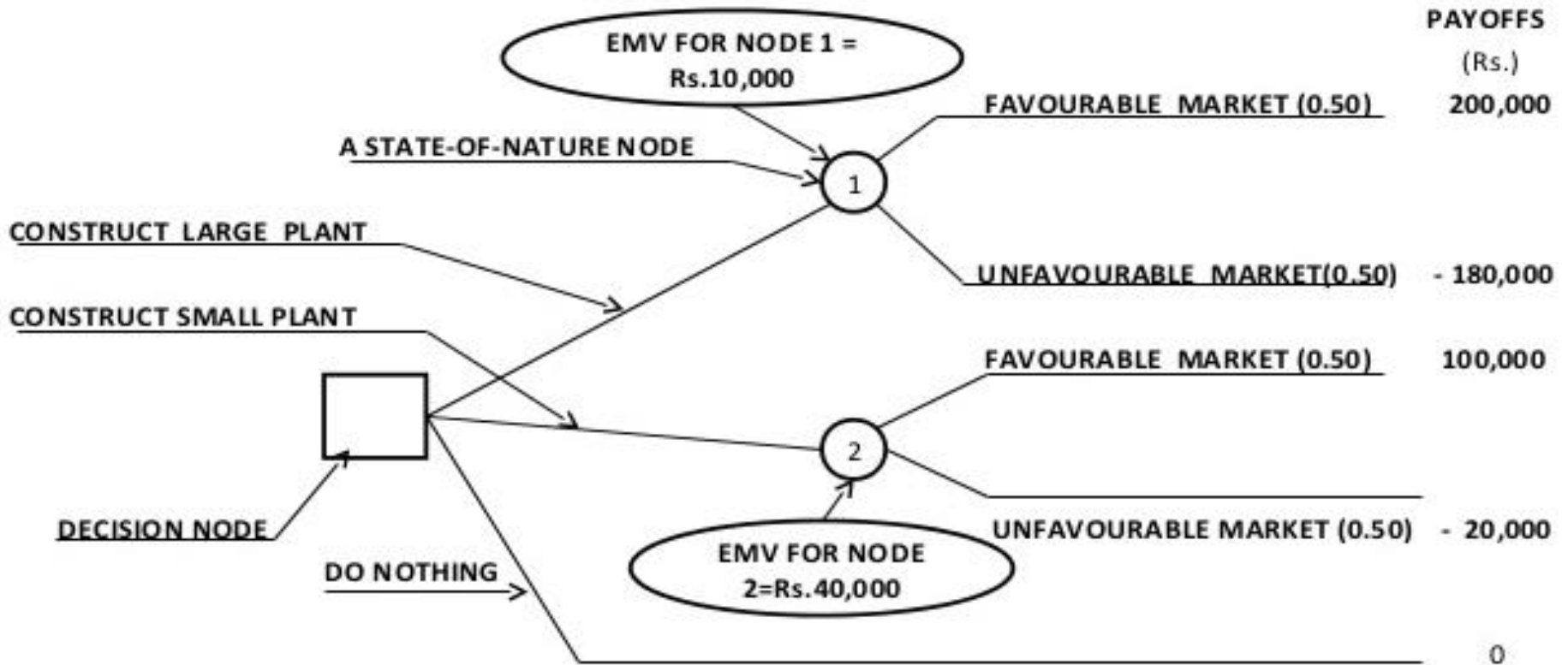
In drawing the tree, we begin at the left and move to the right. Branches from the squares (decision nodes) represent alternatives, and branches from the circles (state-of-nature node) represent the state of nature.

Figure 1 gives the basic decision tree of John Thompson problem.

Five steps of decision tree analysis :

1. **Define the problem**
2. **Draw the decision tree**
3. **Assign probabilities to the state of nature**
4. **Estimate payoffs for each possible combination of alternative and state of nature**
5. **SOLVE THE PROBLEM BY COMPUTING EXPECTED MONETARY VALUES (EMVs) FOR EACH STATE OF NATURE NODE. This is done by starting at the right of the tree and working back to decision node on the left. At each decision node, the alternative with best EMV is selected.**

Decision Tree



EMV FOR NODE 1 = $(0.50) (200,000) + (0.50) (-180,000) = \text{Rs.}10,000$

EMV FOR NODE 2 = $(0.50) (100,000) + (0.50) (-20,000) = \text{Rs.}40,000$

SO, A SMALL PLANT SHOULD BE BUILT